ANSWER KEY TO EMET3004 MIDTERM EXAM

1. (a) Otherwise the relationship b/w X and Y would be deterministic.
   (b) Something like $f(x+1,u) - f(x,u)$
   (c) Change of function value as X increases by 1, starting at x
   (d) β_1; does not differ from average causal effect
   (e) $\text{argmin}_b \sum (Y_i - b \cdot g(X_i))^2$

2. (a) False. $\bar{Y} + 1/n$ is biased but consistent.
   (b) Could be true or false. If $E[u_i] = 0$ then the statement is false, otherwise it is true.
   (c) True. Patients who receive cc constitute a select sample. For example, heart attack patients who pose too great a risk for cc will not even be considered for cc.
   (d) False. Even though we include Z there is still a part of X that is correlated with the original error term u; simply throwing in Z does not change that
   (e) True. Justification: Gauss Markov Theorem.

3. (a) What is the effect of body size on earnings?
   (b) Dependent: log of hourly wage. Independent: height and BMI
   (c) simultaneous causality and ovb (e.g., socio-economic factors), BMI of biological family members
   (d) height has a positive causal effect, BMI seems insignificant
   (e) measurement error in height and BMI

4. (a) $\pi_1 \neq 0$ (relevance), $E[u_i|Z_i] = E[u_i]$ (exogeneity)
   (b)

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\begin{aligned}
\frac{s_{ZY}}{s_Z^2} &:= \frac{\sum(Z_i - \bar{Z})(Y_i - \bar{Y})}{\sum(Z_i - \bar{Z}^2)} \\
&= \frac{\sum(Z_i - \bar{Z})\left(\beta_1(X_i - \bar{X}) + (u_i - \bar{u})\right)}{\sum(Z_i - \bar{Z})^2} \\
&= \frac{\beta_1 \sum(Z_i - \bar{Z})(X_i - \bar{X}) + \sum(Z_i - \bar{Z})(u_i - \bar{u})}{\sum(Z_i - \bar{Z})^2} \\
&= \frac{\beta_1 \sum(Z_i - \bar{Z})\left(\pi_1(Z_i - \bar{Z}) + (v_i - \bar{v})\right) + \sum(Z_i - \bar{Z})(u_i - \bar{u})}{\sum(Z_i - \bar{Z})^2} \\
&= \frac{\beta_1\pi_1 \sum(Z_i - \bar{Z})(Z_i - \bar{Z}) + \beta_1 \sum(Z_i - \bar{Z})(v_i - \bar{v}) + \sum(Z_i - \bar{Z})(u_i - \bar{u})}{\sum(Z_i - \bar{Z})^2} \\
&= \beta_1\pi_1 + \frac{\beta_1 \frac{1}{n} \sum(Z_i - \bar{Z})(v_i - \bar{v}) + \frac{1}{n}\sum(Z_i - \bar{Z})(u_i - \bar{u})}{\frac{1}{n}\sum(Z_i - \bar{Z})^2} \\
&\simeq \beta_1\pi_1 + \frac{\beta_1 \text{Cov}(Z_i, v_i) + \text{Cov}(Z_i, u_i)}{\text{Var}(Z_i)} \\
&= \beta_1\pi_1,
\end{aligned}
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   because Z_i is uncorrelated with both v_i and u_i.
   (c) Obvious: $(s_{ZY}/s_Z^2)/\pi_1$