Advanced Econometrics I
EMET4314/8014
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Juergen Meinecke
Research School of Economics
ANU

Assignment 6
(due: Tuesday week 7, 11:00am)

Submission Instructions: Same as last week.

Exercises

Provide transparent derivations. Justify steps that are not obvious. Use self sufficient proofs. Make reasonable assumptions where necessary.

Let $Y_i = X_i'\beta + e_i$ with $\mathrm{E}(e_i X_i) \neq 0$. You have available Z_i with $\mathrm{E}(e_i Z_i) = 0$ and $\dim Z_i = L \geq \dim X_i = K$. Consider the estimator

$$b_P := \left(\sum_{i=1}^{N} (PZ_i) X_i' \right)^{-1} \left(\sum_{i=1}^{N} (PZ_i) Y_i \right),$$

where $\dim P = K \times L$. Different choices for the matrix P result in different estimators. For example, the simple IV estimator for the exactly identified case simply sets $P = I$. It can be shown that another choice, namely $P = P^* := \mathrm{E}(X_i Z_i')\mathrm{E}(Z_i Z_i')^{-1}$, results in an estimator, b_{P^*}, with minimal asymptotic variance.

Notice, however, that b_{P^*} is an *infeasible* estimator because you do not observe P^*. Replace P^* by $\hat{P} = P^* + \mathrm{o}_p(1)$ resulting in the *feasible* estimator $b_{\hat{P}}$.

(i) Prove that $b_{\hat{P}}$ is consistent.

(ii) Derive the asymptotic distribution of $\sqrt{N}(b_{\hat{P}} - \beta)$.

Provide the asymptotic variance in terms of P^*. Then plug in the right hand side of $P^* := \mathrm{E}(X_i Z_i')\mathrm{E}(Z_i Z_i')^{-1}$ and see how the result simplifies considerably.

You may assume $\mathrm{E}(e_i^2 Z_i Z_i') = \sigma_e^2 \mathrm{E}(Z_i Z_i')$ to make things easier.

Use the following nomenclature for brevity:

$$C_{XZ} := \mathrm{E}(X_i Z_i') \qquad C_{ZX} := \mathrm{E}(Z_i X_i') \qquad C_{ZZ} := \mathrm{E}(Z_i Z_i')$$

This implies $C_{XZ} = C_{ZX}'$ and $P^* = C_{XZ} C_{ZZ}^{-1}$.

(iii) Suggest a good $\hat{P}$ for P^* such that $\hat{P} = P^* + \mathrm{o}_p(1)$.