Advanced Econometrics I
EMET4314/8014
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Assignment 9
(due: Tuesday week 10, 11:00am)

Submission Instructions: Same as last week.

Exercises

Provide transparent derivations. Justify steps that are not obvious. Use self sufficient proofs. Make reasonable assumptions where necessary.

In the lecture we learned about the *potential outcomes framework*. Translated into a regression model, it can be represented by

$$Y_i = \beta_0 + \beta_{1i} X_i + u_i$$
$$X_i = \pi_0 + \pi_{1i} Z_i + v_i,$$

where

$$\beta_0 := \mathrm{E}\left(Y_i(0)\right) \qquad \beta_{1i} := Y_i(1) - Y_i(0) \qquad u_i := Y_i(0) - \mathrm{E}\left(Y_i(0)\right)$$
$$\pi_0 := \mathrm{E}\left(X_i(0)\right) \qquad \pi_{1i} := X_i(1) - X_i(0) \qquad v_i := X_i(0) - \mathrm{E}\left(X_i(0)\right)$$

This looks like a two stage regression in which the slope coefficients are individual-specific. You have iid sample data (X_i, Y_i, Z_i) and compute $\hat{\beta}^{\mathrm{IV}}$. From the week 5 lecture you know

$$\hat{\beta}^{\mathrm{IV}} = \frac{s_{ZY}}{s_{ZX}} = \frac{\sigma_{ZY}}{\sigma_{ZX}} + \mathrm{o}_p(1),$$

where s_{ZY} denotes the sample covariance and σ_{ZY} denotes the population covariance of Z_i and Y_i (and likewise for the objects in the denominator).
The goal here is to present σ_{ZY} and σ_{ZX} in terms of moments of π_{1i} and β_{1i}. In your derivations, please assume random assignment of Z_i so that

$$\big(Y_i(1), Y_i(0), X_i(1), X_i(0)\big) \perp\!\!\!\perp Z_i$$

(i) Prove that $\sigma_{ZX} = \sigma_Z^2 \mathrm{E}(\pi_{1i})$.
(Note: π_{1i} was defined in the lecture)

(ii) Prove that $\sigma_{ZY} = \sigma_Z^2 \mathrm{E}(\beta_{1i}\pi_{1i})$.
(Note: β_{1i} was defined in the lecture)

Putting things together: $\hat{\beta}^{\mathrm{IV}} = \mathrm{E}(\beta_{1i}\pi_{1i})/\mathrm{E}(\pi_{1i}) + \mathrm{o}_p(1)$. We called the probability limit the *local average treatment effect* (LATE).