Advanced Econometrics I
EMET4314/8014
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Juergen Meinecke
Research School of Economics
ANU

Assignment 10
(due: Tuesday week 11, 11:00am)

Submission Instructions: Same as last week.

Exercise

Provide transparent derivations. Justify steps that are not obvious. Use self sufficient proofs. Make reasonable assumptions where necessary.

You have $Y_1, \dots, Y_N$ iid with pdf

$$f(y|\mu) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(y-\mu)^2\right).$$

Notice that this is the pdf of the normal distribution with mean μ and variance 1.

(i) Derive the log likelihood function $\ln L(\mu)$.

(ii) Derive $\hat{\mu}^{\text{ML}}$. Is it unbiased?

(iii) Derive $\text{Var}(\hat{\mu}^{\text{ML}})$.

(iv) Derive the score function $S(y|\mu)$ as it was defined during the lecture.

(v) Derive the Fisher information $I(\mu)$ via $\text{E}\left(S(Y|\mu)^2\right)$. Suppose you have an unbiased estimator $T(Y_1, \dots, Y_N)$ for μ, what is the Cramér Rao lower bound for its variance?

(vi) Confirm the information equality.

(vii) Does $\hat{\mu}^{\text{ML}}$ attain the CRB?

(viii) If $\hat{\mu}^{\text{ML}}$ does indeed attain the CRB, then the average score function can be decomposed as suggested by the Theorem in the subsection *Minimum Variance Unbiased Estimators* of the week 10 slides. Derive that decomposition and provide $a(\mu)$.

(ix) State the asymptotic distribution of $\hat{\mu}^{\text{ML}}$. (No need to derive!)