THE AUSTRALIAN NATIONAL UNIVERSITY

First Semester Final Examination– June, 2019

Advanced Econometrics I

(EMET 4314/8014)

Reading Time: 0 Minutes
Writing Time: 120 Minutes

INSTRUCTIONS

- Answer all 4 questions of this handout
- Write on paper or an electronic device (such as an Ipad). Handwritten answers only! Do not type anything.
- If written on a piece of paper, scan your work and upload to Wattle.
- If using an electronic device, upload your file to Wattle.
- Provide complete, self-contained, and correct answers!
- Always show your work!
- Good luck!

EXAM QUESTIONS

1. [15 marks]

   Let $Y_i = X_i'\beta + u_i$, where $\mathrm{E}(u_i X_i) = 0$. Notice that $\dim X_i = K \times 1$.

   You know that the solution to $\hat{\beta}^{\mathrm{OLS}} := \underset{b \in \mathbb{R}^K}{\operatorname{argmin}} \sum_{i=1}^{N} (Y_i - X_i' b)^2$ is

   $$\hat{\beta}^{\mathrm{OLS}} = \left(\sum_{i=1}^{N} X_i X_i' \right)^{-1} \sum_{i=1}^{N} X_i Y_i.$$

   Note: In your derivations for parts (a), (b), and (c), make use of the stochastic order symbols $\mathrm{o}_p(1)$ and $\mathrm{O}_p(1)$ wherever you can.

   (a) Derive the probability limit of $\hat{\beta}^{\mathrm{OLS}}$.

   (b) Derive the asymptotic distribution of $\sqrt{N}(\hat{\beta}^{\mathrm{OLS}} - \beta)$ assuming homoskedasticity.

   (c) Suggest a consistent estimator for the asymptotic variance of $\sqrt{N}(\hat{\beta}^{\mathrm{OLS}} - \beta)$ under homoskedasticity. Prove that it is consistent.

2. [15 marks]

   Are the following statements true or false? Provide a complete yet short explanation. Use mathematical derivations where necessary.

   (Note: you will not receive any credit without providing a correct explanation.)

   (a) $\mathrm{Cov}(X, Y) = \mathrm{Cov}(X, \mathrm{E}(Y|X))$.

   (b) In the linear model, omitting explanatory variables does not necessarily lead to bias in the OLS estimator.

   (c) Any maximum likelihood estimator is also an M-estimator.

   (d) You have a random sample $Y_1, \ldots, Y_N$ with $N > 3$ and your goal is to estimate $\mathrm{E}(Y_3)$. Consider the estimator $\hat{\theta} := Y_3$. That estimator is unbiased and efficient.

   (e) In a model with heterogeneous slope coefficients, we defined

   $$\mathrm{LATE} := \frac{\mathrm{E}(\beta_{1i} \cdot \pi_{1i})}{\mathrm{E}(\pi_{1i})}.$$

   It can be shown that LATE = ATE.

3. [15 marks]

   You have a random sample $Y_1, \ldots, Y_N$ drawn from a *Poisson* distribution. The probability mass function of Y_i is

   $$f_Y(y|\theta) = \frac{\theta^y \exp(-\theta)}{y!}, \qquad \text{with } y = 0, 1, 2, \ldots$$

   It is known that $\mathrm{E}(Y_i) = \mathrm{Var}\,(Y_i) = \theta$.

(a) Derive the likelihood function $\mathscr{L}(\theta|y_1,\ldots,y_N)$.

(b) Derive the log likelihood function $L(\theta|y_1,\ldots,y_N)$.

(c) Derive the score function $\partial \ln f_Y(y|\theta)/\partial\theta$.

(d) Derive the maximum likelihood estimator $\hat{\theta}^{\text{ML}}$ of θ.

(e) Derive $\text{E}(\hat{\theta}^{\text{ML}})$.

(f) Derive Var $(\hat{\theta}^{\text{ML}})$.

(g) Derive the Cramér Rao bound.

(h) Is $\hat{\theta}^{\text{ML}}$ efficient?

4. [15 marks]

(a) [5 marks]
Consider Heckman's sample selection model

$$\begin{aligned} Y_i^* &= X_i'\beta + e_i \\ D_i &= 1 \cdot \left(Z_i'\gamma + v_i > 0\right), \end{aligned}$$

where, most generally, $X_i \neq Z_i$.
You do not observe Y_i^*. Instead, you observe a random sample (D_i, X_i, Y_i, Z_i) for $i = 1, \ldots, N$, where

$$Y_i := \begin{cases} Y_i^* & \text{if } D_i = 1 \\ \text{unobserved} & \text{if } D_i = 0, \end{cases}$$

and your goal is to estimate β.
A naïve approach here would be to simply regress Y_i on X_i, ignoring possible sample selection. Let $\hat{\beta}^{\text{OLS}}$ be the estimator in a regression of Y_i on X_i. Prove or disprove: $\text{E}\left(\hat{\beta}^{\text{OLS}}|D_i = 1, X_i, Z_i\right) = \beta$.
To obtain useful results, you should impose additional assumptions in your derivation. These additional assumptions should be based on the discussion of the sample selection model in the lecture.

[*Question 4 continues on next page*]

(b) [10 marks]

Consider the *censored regression model*

$$Y_i^* = X_i'\beta + e_i,$$

where the errors e_i are iid draws from $N(0,1)$. Alternatively, this model is called the *standard Tobit model* or *Type 1 Tobit model*.

You do not observe Y_i^*. Instead, you observe a random sample (X_i, Y_i) for $i = 1, \dots, N$, where

$$Y_i := \begin{cases} Y_i^* & \text{if } Y_i^* \geq 0 \\ 0 & \text{if } Y_i^* < 0, \end{cases}$$

and your goal is to estimate β via maximum likelihood.

Write down the log likelihood function $L(\beta | x_1, \dots, x_N, y_1, \dots, y_N)$.

End of Exam