Advanced Econometrics I
EMET4314/8014
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Problem Set 6
(due: Monday week 8, 11:00am)

Submission Instructions: Same as last week.

Exercises

Provide transparent derivations. Justify steps that are not obvious. Use self sufficient proofs. Make reasonable assumptions where necessary.

1. In the lecture we learned about the *potential outcomes framework*. Translated into a regression model, it can be represented by

$$Y_i = \beta_0 + \beta_{1i} X_i + u_i$$
$$X_i = \pi_0 + \pi_{1i} Z_i + v_i,$$

where

$$\beta_0 := \mathrm{E}\left(Y_i(0)\right) \qquad \beta_{1i} := Y_i(1) - Y_i(0) \qquad u_i := Y_i(0) - \mathrm{E}\left(Y_i(0)\right)$$
$$\pi_0 := \mathrm{E}\left(X_i(0)\right) \qquad \pi_{1i} := X_i(1) - X_i(0) \qquad v_i := X_i(0) - \mathrm{E}\left(X_i(0)\right)$$

This looks like a two stage regression in which the slope coefficients are individual-specific.

You have iid sample data (X_i, Y_i, Z_i) and compute $\hat{\beta}^{\mathrm{IV}}$. From the week 5 lecture you know

$$\hat{\beta}^{\mathrm{IV}} = \frac{s_{ZY}}{s_{ZX}} = \frac{\sigma_{ZY}}{\sigma_{ZX}} + \mathrm{o}_p(1),$$

where s_{ZY} denotes the sample covariance and σ_{ZY} denotes the population covariance of Z_i and Y_i (and likewise for the objects in the denominator).

The goal here is to present σ_{ZY} and σ_{ZX} in terms of moments of π_{1i} and β_{1i}. In your derivations, please assume random assignment of Z_i so that

$$\big(Y_i(1), Y_i(0), X_i(1), X_i(0)\big) \perp\!\!\!\perp Z_i$$

(a) Prove that $\sigma_{ZX} = \sigma_Z^2 \mathrm{E}(\pi_{1i})$.
(Note: π_{1i} was defined in the lecture)

(b) Prove that $\sigma_{ZY} = \sigma_Z^2 \mathrm{E}(\beta_{1i}\pi_{1i})$.
(Note: β_{1i} was defined in the lecture)

Putting things together: $\hat{\beta}^{\mathrm{IV}} = \mathrm{E}(\beta_{1i}\pi_{1i})/\mathrm{E}(\pi_{1i}) + \mathrm{o}_p(1)$. We called the probability limit the *local average treatment effect* (LATE).

2. Let $Y_i = X_i'\beta + e_i$ with $\mathrm{E}(e_i X_i) \neq 0$. You have available Z_i with $\mathrm{E}(e_i Z_i) = 0$ and dim $Z_i = L \geq$ dim $X_i = K$. Consider the estimator

$$b_P := \left(\sum_{i=1}^{N} (PZ_i) X_i' \right)^{-1} \left(\sum_{i=1}^{N} (PZ_i) Y_i \right),$$

where dim $P = K \times L$. Different choices for the matrix P result in different estimators. For example, the IV estimator for the exactly identified case sets $P = I$.

It can be shown that another choice, namely $P = P^* := \mathrm{E}(X_i Z_i')\mathrm{E}(Z_i Z_i')^{-1}$, results in an estimator, b_{P^*}, with minimal asymptotic variance.

Notice, however, that b_{P^*} is an *infeasible* estimator because you do not observe P^*. Replace P^* by $\hat{P} = P^* + \mathrm{o}_p(1)$ resulting in the *feasible* estimator $b_{\hat{P}}$.

(a) Prove that $b_{\hat{P}}$ is consistent.

(b) Derive the asymptotic distribution of $\sqrt{N}(b_{\hat{P}} - \beta)$.
Provide the asymptotic variance in terms of P^*. Then plug in the right hand side of $P^* := \mathrm{E}(X_i Z_i')\mathrm{E}(Z_i Z_i')^{-1}$ and see how the result simplifies considerably.
You may assume $\mathrm{E}(e_i^2 Z_i Z_i') = \sigma_e^2 \mathrm{E}(Z_i Z_i')$ to make things easier.
Use the following nomenclature for brevity:

$$C_{XZ} := \mathrm{E}(X_i Z_i') \qquad C_{ZX} := \mathrm{E}(Z_i X_i') \qquad C_{ZZ} := \mathrm{E}(Z_i Z_i')$$

This implies $C_{XZ} = C_{ZX}'$ and $P^* = C_{XZ} C_{ZZ}^{-1}$.

(c) Suggest a good $\hat{P}$ for P^* such that $\hat{P} = P^* + \mathrm{o}_p(1)$.

3. Read the paper "*A Practical Guide to Weak Instruments*" by Keane and Neal, published 2024 in the Annual Review of Economics.

Try to read the whole paper and make sense of as much as you possible. I don't expect you to understand everything, but hopefully you will be able to follow the main points. The objective is to prepare you for the week 8 lecture in which we will discuss the paper in more detail.