THE AUSTRALIAN NATIONAL UNIVERSITY

First Semester Final Examination– June, 2022

Advanced Econometrics I

(EMET 4314/8014)

Reading Time: 0 Minutes
Writing Time: 120 Minutes

Instructions

- Answer all 4 questions of this handout.
- Write on paper or an electronic device (such as an Ipad).
- Handwritten answers only! Do not type anything.
- Work on your answers only between 12:00pm–2:00pm.
- If written on a piece of paper, scan your work and upload to Wattle.
- If using an electronic device, upload your file to Wattle.
- Upload your file as soon as possible after 2:00pm.
  You have at most 15 minutes to scan and upload your document.
- Provide complete, self-contained, and correct answers.
- Make reasonable assumptions where necessary.
- Justify all steps that are not obvious.

`Beginning of Exam Questions`

1. `[1 mark total]` Write the following statement by hand:

   I hereby declare

   - *to uphold the principles of academic integrity, as defined in the University Academic Misconduct Rules;*
   - *that your work in the final exam in no part involves copying, cheating, collusion, fabrication, plagiarism or recycling.*

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2. `[20 marks total]`

   Consider the scalar model $Y_i = \beta_0 + \beta_1 X_{i1} + e_i$ where $e_i | X_{i1} \sim \mathcal{N}(0, 1)$.

   You have available a random sample (X_{i1}, Y_i), $i = 1, \ldots, N$.

   Let $\widehat{\beta}_0$ and $\widehat{\beta}_1$ be the OLS estimators obtained from a regression of Y_i on a constant and X_{i1}.

   (a) `[2 marks]` State $\widehat{\beta}_1$ in terms of sample moments of the data (that is, sample means, variances, and covariances). No derivation, just state the result.

   (b) `[2 marks]` Derive $\text{Var}\left(\widehat{\beta}_1 | X_{i1}\right)$.

   You observe an additional variable, X_{i2}. Denote by $\widehat{\pi}_0$ and $\widehat{\pi}_1$ the OLS estimators from a regression of X_{i1} on a constant and X_{i2}. Define $\widehat{X}_{i1} := \widehat{\pi}_0 + \widehat{\pi}_1 X_{i2}$.

   Let $\widehat{\theta}_0$ and $\widehat{\theta}_1$ be the OLS estimators obtained from a regression of Y_i on a constant and $\widehat{X}_{i1}$.

   (c) `[4 marks]` Derive $\widehat{\theta}_1$ in terms of sample moments of the data.

   (d) `[2 marks]` Derive $\text{Var}\left(\widehat{\theta}_1 | X_{i1}, X_{i2}\right)$.

   (e) `[5 marks]` Prove or disprove: $\widehat{\theta}_1 = \beta_1 + o_p(1)$.

   (f) `[5 marks]` Which estimator do you prefer: $\widehat{\beta}_1$ or $\widehat{\theta}_1$? Why?

3. [20 marks total - 5 marks each]

Are the following statements true or false? Provide a complete explanation. Use mathematical derivations where necessary.

(Note: you will not receive any credit without providing a correct explanation.)

(a) Let the discrete random variable have the following distribution:
$P(Y = 1) = \pi_1, \qquad P(Y = 2) = \pi_2, \qquad P(Y = 3) = \pi_3,$
where $\pi_1 \in (0, 1)$, $\pi_2 \in (0, 1)$, $\pi_3 \in (0, 1)$, and $\pi_1 + \pi_2 + \pi_3 = 1$.
In a random sample of size N you observe N_1 realizations for which $Y = 1$, N_2 realizations for which $Y = 2$, N_3 realizations for which $Y = 3$, so that $N_1 + N_2 + N_3 = N$.
Then the maximum likelihood estimate of π_1 is N_1/N.

(b) Let X be a Bernoulli random variable, that is, $X = 1$ with probability π and $X = 0$ with probability $1 - \pi$ where $\pi \in (0, 1)$. Let Y be another random variable (not Bernoulli distributed) and assume that $\mathrm{Cov}(X, Y) \neq 0$.
Then $\mathrm{Cov}(X, XY) = \mathrm{E}(Y) + (1 - \pi) \cdot \mathrm{Cov}(X, Y)$.

(c) Let the random variable Z be such that $\mathrm{E}(Z) = 3$ and $\mathrm{E}(Z^2) = 13$. Then a lower bound for $P(-2 < Z < 8)$ is given by $21/25$.

Note: part (d) on next page …

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(d) The Monte Carlo simulation of the simple schooling model from week 7, as summarized by the `Julia` code and corresponding output below, illustrates that the OLS estimator is a consistent estimator for the return to schooling.

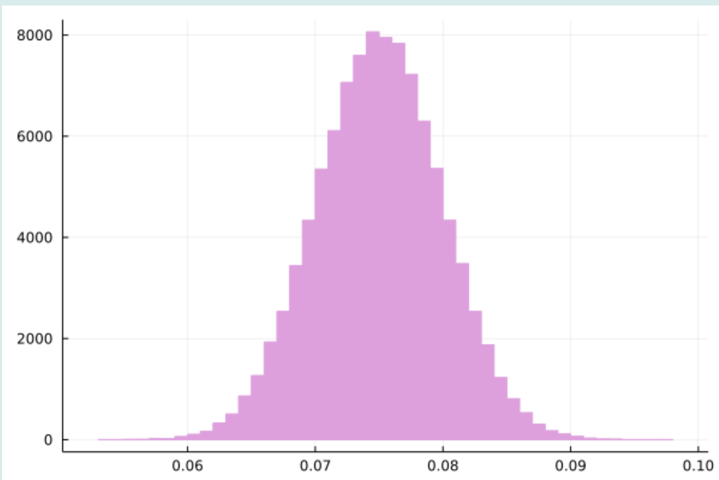
Julia Code

```julia
using Distributions, Random, Plots

function schooling_sample(b2, n;
            p=13.2, b1=4.7, b3=0,
            su=0.175, sa=7.2)
    u = rand(Normal(0, sqrt(su)), n)
    a = rand(Normal(0, sqrt(sa)), n)
    S = p .+ a
    Y = b1 .+ b2*S .+ b3*a .+ u
    return S, Y
end

rep = 100000
b2 = Array{Float64}(undef, rep)
for r in 1:rep
    n = 1000
    x, y = schooling_sample(0.075, n)
    b1_tmp, b2_tmp= [ones(n, 1) x]\y
    b2[r] = b2_tmp
end
histogram(b2, normed = false)
```

Output



4. [20 marks total]

Consider the model

$$Y_i = \mu(X_i, \theta) + e_i, \qquad \text{where } e_i | X_i \sim \mathcal{N}(0, \sigma_e^2).$$

The variables Y_i and e_i are scalars and $\dim(X_i) = K \times 1$ and $\dim(\theta) = L \times 1$ where $K \neq L$. The functional form of μ is considered *known* but is left unspecified here.

You have available a random sample (X_i, Y_i), $i = 1, \ldots, N$, to estimate the unknown parameters θ and the scalar σ_e^2.

(a) [3 marks] Derive the conditional log likelihood function $L(\theta, \sigma_e^2)$.

(b) [3 marks] Derive the score function.

(c) [3 marks] Derive the expected value of the score conditional on X_i.

(d) [2 marks] Determine the MLE of σ_e^2 as a function of $\widehat{\theta}^{\text{ML}}$ (the MLE of θ).

(e) [3 marks] Derive the Hessian matrix as the derivative of the score.

(f) [6 marks] Show that the information equality holds here.

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End of Exam