Advanced Econometrics I
EMET4314/8014
Semester 1, 2024

Juergen Meinecke
Research School of Economics
ANU

Assignment 5
(due: Tuesday week 6, 11:00am)

Submission Instructions: Same as last week.

The solutions will be discussed in the Friday workshop during week 5. Please let me know which exercises I should focus on.

Exercises

Provide transparent derivations. Justify steps that are not obvious. Use self sufficient proofs. Make reasonable assumptions where necessary.

1. Remember the following parameter from the week 5 lecture:

$$\pi = \mathrm{E}(Z_i Z_i')^{-1}\mathrm{E}(Z_i X_i'),$$

where $X_i = (X_{i1}', X_{i2}')'$ and $Z_i = (X_{i1}', Z_{i2}')'$. I claimed that π is an upper triangular block matrix. Show this!

Make use of the following result for the inverse of *partitioned* (or block) matrices (taken from Greene's textbook):

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^{-1} = \begin{pmatrix} A_{11}^{-1}(I + A_{12}F_2A_{21}A_{11}^{-1}) & -A_{11}^{-1}A_{12}F_2 \\ -F_2A_{21}A_{11}^{-1} & F_2 \end{pmatrix},$$

where $F_2 := \left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1}$.

2. Let $Y_i = X_i\beta + e_i$ where X_i is a scalar random variable and $\mathrm{E}(e_i|X_i) = 0$. You observe $(\widetilde{X}_i, Y_i)$ with $\widetilde{X}_i := X_i + r_i$ where r_i is a random error. Derive the probability limit of the OLS estimator in the regression of Y_i on $\widetilde{X}_i$. For simplicity, assume that $\mathrm{E}X_i = \mathrm{E}r_i = 0$.

What is the probability limit equal to when the variance of r_i is zero? When the variance of r_i is very large? What is the intuition here?

Hint: Your probability limit should have the form $\beta(1 - \textit{stuff})$, where *stuff* depends only on the population variances of r_i and X_i.

Note: This phenomenon is usually referred to as *measurement error bias* or *attenuation bias*. It is not good to use the word *bias* here though. After all, we are not studying the expected value of the OLS estimator, instead we are studying its probability limit and show that it does not equal β. Biasedness and inconsistency are not the same thing.

3. Prove that $\hat{\beta}^{\mathrm{IV}}$ is consistent.

4. Derive the asymptotic distribution of $\sqrt{N}(\hat{\beta}^{\mathrm{IV}} - \beta)$.