Australian National University

Venue _____

Student Number | | | | | | | | |

Research School of Economics

PRACTICE EXAMINATION

Semester 1 – End of Semester, 2026

EMET4314/EMET8014_Semester 1 Advanced Econometrics

This paper is for ANU students only.

Examination Duration: 180 minutes
Reading Time: 0 Minutes

Exam Conditions:
N/A

Materials Permitted In The Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)
No materials permitted

Materials To Be Supplied To Students:
1 x 20 page

Instructions To Students:

- There are 3 exam questions, each counting 20 raw marks.
- Answer all exam questions.
- Provide careful, complete, and self-contained proofs.
- Make reasonable assumptions where necessary.
- Justify all steps that are not obvious.

Question 1

Consider the model $Y_i = X_i'\beta + e_i$, where $\mathrm{E}(X_i e_i) \neq 0$, and X_i has dimension $K > 1$. You have available a vector Z_i of dimension $L > K$ that satisfies $\mathrm{E}(Z_i e_i) = 0$.

Study the following estimator (which you know from lecture):

$$b = \left(\left(\sum_{i=1}^{N} X_i Z_i' \right) \left(\sum_{i=1}^{N} Z_i Z_i' \right)^{-1} \left(\sum_{i=1}^{N} Z_i X_i' \right) \right)^{-1} \left(\left(\sum_{i=1}^{N} X_i Z_i' \right) \left(\sum_{i=1}^{N} Z_i Z_i' \right)^{-1} \left(\sum_{i=1}^{N} Z_i Y_i \right) \right).$$

(a) `[8 marks]` Prove that b is consistent.

(b) `[8 marks]` Derive the asymptotic distribution of the estimator b.

(c) `[4 marks]` Which estimator would you propose if $K = L = 1$? Use your result in part (b) to derive a simpler expression for the asymptotic variance.

Question 2

Are the following statements true or false? Provide a complete explanation.
(Note: you will not receive any credit without providing a correct explanation.)

(a) `[5 marks]`

`Statement`: For a random variable X, if $\mathrm{E}(X) < \infty$, then $\mathrm{E}(X^2) < \infty$.

(b) `[5 marks]`

`Statement`: Let c be a real number. Then the sequence $\frac{c}{N} = \mathrm{o}_p(1)$, for $N = 1, 2, 3, \ldots$.

(c) `[5 marks]` You have a random sample $Y_1, \ldots, Y_N$ and want to estimate $\mathrm{E}(Y_1)$. Consider the following two estimators:

$$\widehat{\mu} := \underset{m \in \mathbb{R}}{\operatorname{argmin}} \sum_{i=1}^{N} (Y_i - m)^2$$
$$\widetilde{\mu} := \tfrac{1}{N} \left(\tfrac{1}{2} Y_1 + \tfrac{3}{2} Y_2 + \tfrac{1}{2} Y_3 + \tfrac{3}{2} Y_4 + \cdots + \tfrac{1}{2} Y_{N-1} + \tfrac{3}{2} Y_N \right)$$

`Statement`:

Both estimators are unbiased and have the same variance.
(Note: You may assume that N is an even number.)

(d) `[5 marks]` Let X and Z be two independent standard normally distributed random variables and let $Y = X^2 + Z$. Consider β_1 in $Y = \beta_0 + \beta_1 X + u$, where $\mathrm{E}(uX) = 0$.

`Statement`: It follows that $\beta_1 = 0$.

Question 3

Consider the model

$$Y = \mu(X, \theta) + e,$$

where the variables Y and e are scalars and $\dim(X) = K \times 1$ and $\dim(\theta) = M \times 1$ where K is not necessarily equal to M. The symbol μ is a placeholder for a function that is twice continuously differentiable in θ.

The errors e are independently distributed of the X, each with density

$$f(e) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}e^2\right).$$

(a) [2 marks] Derive the conditional density function $f_{Y|X}(y \mid x, \theta, \sigma^2)$.

(b) [2 marks] Derive the score function $S(y \mid x, \theta, \sigma^2)$.

(c) [4 marks] Derive the expected value of the score conditional on $X = x$, that is, $\mathrm{E}\left(S(Y \mid X, \theta, \sigma^2) \mid X = x\right)$.

(d) [4 marks] Determine the MLE of σ^2 as a function of $\widehat{\theta}^{\mathrm{ML}}$ (the MLE of θ).

(e) [4 marks] Derive the Hessian matrix

$$H(y \mid x, \theta, \sigma^2) := \begin{pmatrix} \frac{\partial^2 \ln f_{Y|X}}{\partial\theta\partial\theta'}(y|x, \theta, \sigma^2) & \frac{\partial^2 \ln f_{Y|X}}{\partial\theta\partial\sigma^2}(y|x, \theta, \sigma^2) \\ \frac{\partial^2 \ln f_{Y|X}}{\partial\sigma^2\partial\theta'}(y|x, \theta, \sigma^2) & \frac{\partial^2 \ln f_{Y|X}}{\partial(\sigma^2)^2}(y|x, \theta, \sigma^2) \end{pmatrix}.$$

(f) [4 marks] Derive the expected value of the Hessian conditional on X, that is, $\mathrm{E}\left(H(Y|X, \theta, \sigma^2) \mid X = x\right)$.

End of Exam Questions